tion exists for a vertical potentiometer, where I_x is the average current in the horizontal potentiometers adjacent to it, and I_y is the current flowing through that particular vertical potentiometer. The current components in the vertical direction I_y are usually insignificant and can be neglected except near the corner of the notch. Since the main interest was in the doubled potentiometers across the centerline of the notch, more attention was paid to this area. The value of one potentiometer would be changed and it could be observed in the field-intensity resistance diagram, how this affected the other potentiometers. This amounted to a trial and error process. The problem was solved when all potentiometers fell exactly on the field-intensity resistance curve. This arrangement was never achieved, and the problem was assumed solved when the potentiometers were close to the curve. Any further adjustment of any potentiometer caused a divergence. This divergence was relative in that changing the value of a potentiometer caused some potentiometers to converge and others to diverge. Careful plotting was necessary to see how the overall system of potentiometers was acting and to find the values where no further change in any of the potentiometers was necessary. Through all adjustments the boundary conditions remained constant. Figure 2 shows a typical example of the values of the potentiometers obtained on the d.c.-board for the ratio of $2b/a = 0.2$.

The value of the load L acting on the bar that the resistances on the d.c.-board represented was calculated by

$$L = k\Sigma[\log(R/R_*) + 2]$$

along the symmetry line that was derived from Eq. (9) of the previous paper.[1] The constraint factors for notched bars of $a/d = \frac{5}{7}$ obtained from the d.c.-board are compared in Fig. 3 with theoretical results and measured values obtained by actual testing of a specimen. In the calculation of theoretical lower bounds for the constraint factor, the truncated wedge stress field introduced by Prager and Hodge[3] was used. The actual testing of the specimens was done on a 60,000-lb Tinius Olsen testing machine. Aluminum stock (Alcoa 6061-T6510) with a cross section of $\frac{3}{4} \times 2$ in. was used. The bars were loaded to 50,000 lb (slightly above the yield limit), before the notches were machined into them. After being notched the bars were again placed into the testing machine, and load vs strain curves were taken. The determination of $L/2ak$ was made in the following manner. L was taken as the collapse point for the notched bar on the load vs strain curve. It was determined by the method suggested by Watts and Ford.[4] The 50,000 lb that was initially placed on the unnotched bar was multiplied by the ratio of a/d and this value was used as $2ak$.

In concluding the present report, the authors would like to say that the d.c.-board method has an advantage over the lower bounds found by a truncated wedge stress field, and this is especially true in problems that have complex configurations in which it is difficult to approximate the boundary between the elastic and inelastic regions.

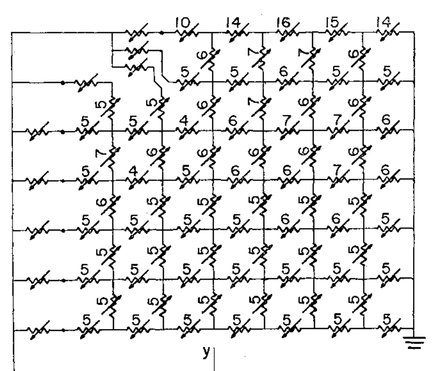

Fig. 2 Potentiometer values obtained by the d.c.-board theory in kilohms for $2b/a = 0.2$.

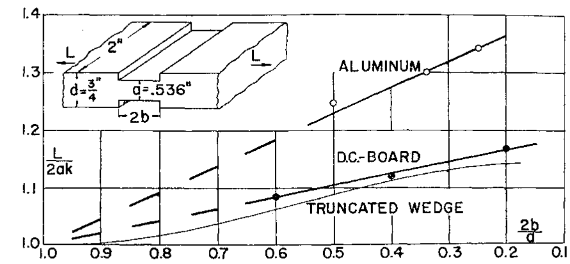

Fig. 3 Comparison of constraint factor of notched bars as obtained by d.c.-board (lower bound), truncated wedge stress field (lower bound), and actual testing.

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Thermal Choking of Partially Ionized Gases

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Thermal choking is a well-known phenomenon in classical aerodynamics.[1] More recently, Resler and Sears[2, 3] investigated analytically channel flows with magnetogasdynamic effects and found increased possibilities of choking and smooth passage at the sonic speed. The purpose of this note is to show that for partially-ionized monatomic gases, a simple criterion such as "choking at Mach number equal to 1" cannot be obtained. Only equilibrium flows of monatomic gases with ionization fraction given by the Saha equation[4] will be considered.

The basic equations are

$$\frac{d}{dx}(nAu) = 0 \tag{1}$$

$$nu\frac{du}{dx} + R\frac{d}{dx}\{n(1+\alpha)T\} = 0 \tag{2}$$

$$nu^2\frac{du}{dx} + \frac{\gamma R}{\gamma - 1}nu\frac{d}{dx}\{(1+\alpha)T\} + nuRT_{\text{ion}}\frac{d\alpha}{dx} = q \tag{3}$$

$$\frac{\alpha}{1-\alpha^2}\frac{p}{p_0} = 3.16 \times 10^{-7}T^{2.5}\exp\left(-\frac{T_{\text{ion}}}{T}\right) \tag{4}$$

$$p = k(1+\alpha)nT \tag{5}$$

where n is the number density of the neutral monatomic gas, α is the number fraction of ionization, A is the area of the channel, q is the rate of heat energy density supplied to the gas, p_0 is the pressure of 1 atm, and T_{ion} is the ionization

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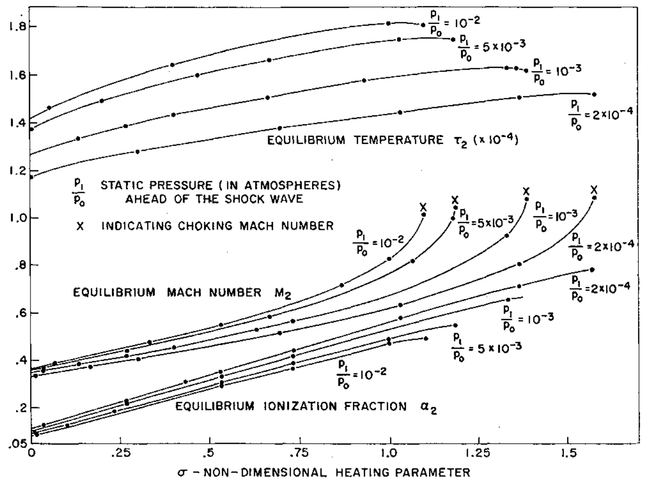

Fig. 1 Temperature, Mach number, and ionization fraction at equilibrium behind shock wave for argon, $M_1 = 15$.

potential of the gas in degrees Kelvin. Other notations in the forementioned equations have their usual meaning.

In the present analysis, q is assumed to be explicitly independent of the derivatives of n, u, A, T, and α with respect to x. An example is the case of Ohmic heating. This implies that the condition for thermal choking does not depend explicitly on q.

Two Mach numbers may be introduced:

$$M = u(3m/5kT)^{1/2} \quad (6)$$

$$M_p = u\{3m/5k(1+\alpha)T\}^{1/2} \quad (7)$$

where m is the mass of the neutral gas molecule, M refers to the speed of sound of the neutral gas, whereas $\{5k(1+\alpha)T/3m\}^{1/2}$ is essentially the sound speed of the ionized gas as a whole.

From the Saha equation, Eq. (4), it is found

$$\frac{2-\alpha}{\alpha(1-\alpha)}\frac{d\alpha}{dx} = \frac{1}{u}\frac{du}{dx} + \left(1.5 + \frac{T_{\text{ion}}}{T}\right)\frac{1}{T}\frac{dT}{dx} \quad (8)$$

By using Eqs. (1–3 and 8),

$$\left\{M_p^2[1+(\gamma-1)(1.5+t_i)F(t_i,\alpha,\chi)] - \left[1+\frac{\gamma-1}{\gamma}(2.5+t_i)F(t_i,\alpha,\chi)\right]\right\}\frac{1}{u}\frac{du}{dx} = \left\{1+\frac{\gamma-1}{\gamma}(1.5+t_i)F(t_i,\alpha,\chi)\right\}\frac{1}{A}\frac{dA}{dx} - \frac{q}{\gamma R(1+\alpha)mnuT} \quad (9)$$

where $t_i = T_{\text{ion}}/T$, $\chi = 2(1-\alpha)/(2-\alpha)$, and

$$F(t_i,\alpha,\chi) = \frac{t_i\alpha\chi}{2+(2+1.5\chi+\chi t_i)\alpha}$$

Equation (9) shows that in a straight duct (A = const), an addition of heat to a subsonic flow ($M_p \ll 1$) always causes the Mach number M_p to rise. A limit will be reached, however, when the factor of (du/udx) in Eq. (9) vanishes. Hence, choking of the flow occurs when

$$M_p^2\{1+(\gamma-1)(1.5+t_i)F(t_i,\alpha,\chi)\} = 1 + [(\gamma-1)/\gamma](2.5+t_i)F(t_i,\alpha,\chi) \quad (10)$$

This expression reduces to the well-known formula $M = 1$, when $\alpha = 0$. It reduces also to the simple form

$$M_p = 1 \quad (11)$$

when $\alpha = 1$ (fully-ionized gases). In general, the value of M_p depends on the ionization fraction and the function T_{ion}/T and is found to be generally close to, but not exactly equal to, one. For $\gamma = \frac{5}{3}$, M_p^2 is found to be generally less than one and is equal to one only for $\alpha = 0$ and 1. Equation (10) suggests the possibility of defining a Mach number, which becomes one at choking for $0 \leq \alpha \leq 1$. Its physical meaning is, however, obscure.

Consider the thermal choking of the flow behind a normal shock wave. Let subscript 1 refer to conditions of the neutral monatomic gas ($\alpha = 0$) ahead of the shock wave and subscript 2 to the equilibrium conditions of the partially-ionized gas behind the shock wave. The hydrodynamic conservation equations are (with $\gamma = \frac{5}{3}$)

$$n_1u_1 = n_2u_2 \quad (12)$$

$$mn_1u_1^2 + n_1kT_1 = mn_2u_2^2 + n_2(1+\alpha_2)kT_2 \quad (13)$$

$$\tfrac{1}{2}mu_1^2 + \tfrac{5}{2}kT_1 + H = \tfrac{1}{2}mu_2^2 + (1+\alpha_2)\tfrac{5}{2}kT_2 + \alpha_2kT_{\text{ion}} \quad (14)$$

where H is the rate of energy density addition to the gas. In nondimensional form, Eqs. (12) and (13) can be written as

$$\tfrac{5}{3}M_1^2 + 1 = [\tfrac{5}{3}M_2^2 + (1+\alpha_2)](M_1/M_2)\tau_2^{1/2} \quad (15)$$

$$\tfrac{1}{3}M_1^2 + 1 + (2M_1^2/3)\sigma = [\tfrac{1}{3}M_2^2 + (1+\alpha_2)]\tau_2 + \tfrac{2}{5}\alpha_2\tau_i \quad (16)$$

where $\tau_2 = T_2/T_1$, $\tau_i = T_{\text{ion}}/T_1$, and $\sigma = H/mu_1^2$ is a nondimensional heating parameter.

Calculations have been carried out for M_2, τ_2, and α_2 by using Eqs. (15, 16, and 4). The results for argon (T_{ion} = 182,000°K) are shown in Fig. 1 for various values of p_1/p_0 and σ. The choking Mach numbers M_2 also are shown.

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Thermal and Electrical Properties of Thin-Film Resistance Gages Used for Heat Transfer Measurement

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Calibration data are given for the variation of the thermal and electrical properties of a thin-film heat transfer gage in the range from 70 to 1000°F. The gage comprises a platinum-alloy film mounted on a pyrex substrate. Calibration is accomplished by the electrical pulsing technique.

An experimental study of base heating of rocket vehicles[1] necessitated the development of a thin-film resistance thermometer gage operable at steady-state temperatures

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